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Closed-Form Performance Analysis of Dual Polarization Based MIMO System in Shadowed-Rician Fading LMS Channels

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Abstract
In this paper, the problem of dual polarization based MIMO Processing in Shadowed-Rician (SR) fading land mobile satellite (LMS) channels is studied. It is shown in literature that polarization is used as a interference rejection method; and, most of the existing analytical results are not in closed-form. In this paper, we derive the closed-form expressions of the moment generating (MGF) function of the received signal-to-noise ratio of the MRC based receiver in SR fading LMS channels. Then we provide closed-form expressions of the symbol error rate (SER) by using MGF approach. The analytical diversity order and capacity of the considered scheme is also derived. It is shown by by derived closed-form capacity expression that the capacity of the considered dual polarization based scheme is improved; and it is found very useful in practical satellite communication systems.
Index Terms: Land mobile satellite (LMS) channel, M-ary phase shift keying (M -PSK), Polarization, M -ary quadrature amplitude modulation (M -QAM), Shadowed-Rician fading.

I. INTRODUCTION

Satellite systems are very useful in broadcasting, disaster relief, and navigation. These systems are also beneficial in under-populated areas [1]. Multiple antenna b ased techniques are useful for improving the performance of the satellite based communication system [2]. By using multiple antennas at the earth station and using maximal ratio combining (MRC) the quality of signal reception can be improved. But it is difficult to install multiple antennas at the satellite due to involvement of high cost in designing the satellite system, in practice. Therefore, dual polarization based approach makes this system more useful in terms of available bandwidth. Two independent signals can be transmitted on the same frequency band by means of two orthogonal polarizations. Hence, we can say that the two orthogonal polarizations can be viewed as two data streams transmitting simultaneously and allowing to double the available spectrum. In [3], the Shadowed-Rician (SR) channel model is proposed which describes very accurately the land mobile satellite (LMS) channel and yields significantly less computational burden as compared to other LMS channel models. The MRC scheme for SR fading based LMS channels has been analyzed in [4]– [7]; nevertheless, most of the analytical results are provided in the form of infinite power series, which are not in closed-form. Whereas, closed-form analytical expressions are more suitable for understanding the characteristics and practical implementation of a communication system. Recently, hybrid satellite-terrestial communication system is discussed in [8]–[13], [21]. Another useful model for the LMS channel is recently discussed in [7] by using the $\kappa - \mu$ shadowed random variables. This model is more accurate for modelling the satellite links than S-R fading but has a complicated form. Because multiple antenna based terrestrial communication systems are widely utilized [25]-[30], therefore it is useful to study the satellite communication system with multiple antennas.

In this paper, we provide closed-form expressions of the moment generating function (MGF) of the received signal-to-noise ratio (SNR) of the MRC scheme based satellite communication system, over SR fading LMS channels. By using these expressions, we get the bit error rate (SER), diversity order, and capacity of the scheme.

II. SYSTEM MODEL

We consider a satellite, with a single antenna, transmitting signals to an earth station containing N receive antennas. The data stream at the satellite is transmitted in two parts; one part uses left-hand circular polarization and other part uses right-hand circular polarization. The signals received at the receiving-end can be written as

$$\mathbf{y}_l = \mathbf{h}_l x_l + \mathbf{e}_l, \tag{1}$$

and

$$\mathbf{y}_r = \mathbf{h}_r x_r + \mathbf{e}_r, \tag{2}$$

where$\mathbf{h}_l \in \mathbb{C}^{N\times 1}$ and $\mathbf{h}_r \in \mathbb{C}^{N\times 1}$ contain the channel gains between left-hand and right-hand feed of the transmit antenna and receive antennas, respectively; x_l and x_r, with E_s average power, are the signals transmitted by the satellite; and $\mathbf{e}_l \in \mathbb{C}^{N\times 1}$ and $\mathbf{e}_r \in \mathbb{C}^{N\times 1}$ denotes the complex-valued additive white Gaussian noise with zero-mean and σ^2 variance. The channel vectors $\mathbf{h}_l$ and $\mathbf{h}_r$ with independent and identically distributed (i.i.d.) SR fading entries can be modeled as $\mathbf{h}_i = \bar{\mathbf{h}} + \tilde{\mathbf{h}}$,

$i = l, r$. Adopting the SR model proposed in [3], the entries of the line-of-sight component $\bar{\mathbf{h}}$ can be modeled as i.i.d. Nakagami-m random variables (RVs) with average power Ω, where m describes the severity of shadowing varying over the range $m \geq 0$. The entries of the scattered component $\tilde{\mathbf{h}}$ are i.i.d. complex Gaussian RVs with zero-mean and unit variance. It is assumed that the receiver contains perfect information of the channel $\mathbf{h}_l$ and $\mathbf{h}_r$. The received SNR of polarization based scheme at the receiver can be written from (1) and (2) as [14]

$$\gamma_l = \frac{\|\boldsymbol{h}_l\|^2}{\sigma^2} E_s, \tag{3}$$

and

$$\gamma_r = \frac{\|\boldsymbol{h}_r\|^2}{\sigma^2} E_s, \tag{4}$$

where $\|\cdot\|^2$ denotes the Euclidean norm.

III. STATISTICS OF THE SNR γ_l

The closed-form expressions of the PDF and CDF of the received SNR γ_l or γ_r are presented in this section. Since the distribution of $\|\mathbf{h}_l\|^2$ and $\|\mathbf{h}_r\|^2$ is same, therefore, we consider only γ_l for the analysis. Let us re-write γ_l as γ, given in (3), as $\gamma = \|\mathbf{h}_l\|^2 \bar{\gamma}$, where $\bar{\gamma} = E_s/\sigma^2$.

$$f_\gamma(x) = \alpha^N \sum_{l=0}^{c} \binom{c}{l} \beta^{c-l} \times (\mathcal{F}(x,l,d,\bar{\gamma}) + \epsilon\delta\mathcal{F}(x,l,d+1,\bar{\gamma})), \tag{5}$$

where

$$\mathcal{F}(x,l,d,\bar{\gamma}) = \frac{(\beta-\delta)^{\frac{l-d}{2}}}{\bar{\gamma}^{\frac{d-l}{2}} \Gamma(d-l)} x^{\frac{d-l}{2}-1} e^{-\frac{\beta-\delta}{2\bar{\gamma}}x} \times M_{\frac{d+l}{2}, \frac{d-l-1}{2}}\left(\frac{\beta-\delta}{\bar{\gamma}}x\right), \tag{6}$$

where $\alpha = 0.5(2bm/(2bm+\Omega))^m/b$, $\beta = (0.5/b)$, $\delta = 0.5\Omega/(2b^2m + b\Omega)$, $2b$ is the average power of the multipath component, $c = (d-N)^+$, $\epsilon = mN - d$, $d = \max\{N, \lfloor mN \rfloor\}$, $\lfloor z \rfloor$ denotes the largest integer not greater than z; $(z)^+$ indicates that if $z \leq 0$, then use $z = 0$; $\max\{\cdot,\cdot\}$ chooses greatest of the two positive integers; $\Gamma(\cdot)$ denotes the Gamma function, and $M_{\mu,\nu}(\cdot)$ represents the Whittaker function.

Lemma 1. The CDF of γ is given by

$$F_\gamma(x) = \alpha^N \sum_{l=0}^{c} \binom{c}{l} \beta^{c-l} \times (\mathcal{G}(x,l,d,\bar{\gamma}) + \epsilon\delta\mathcal{G}(x,l,d+1,\bar{\gamma})), \tag{7}$$

where

$$\mathcal{G}(x,l,d,\bar{\gamma}) = \frac{(\beta-\delta)^{\frac{l-d-1}{2}}}{\bar{\gamma}^{\frac{d-l-1}{2}} \Gamma(d-l+1)} x^{\frac{d-l-1}{2}} e^{-\frac{\beta-\delta}{2\bar{\gamma}}x} \times M_{\frac{d+l-1}{2}, \frac{d-l}{2}}\left(\frac{\beta-\delta}{\bar{\gamma}}x\right), \tag{8}$$

where $N = 2 \times M$, and M is the number of receive antennas.

Proof. Lemma 1 can be proved by using (5), relation $F_\gamma(x) = \int_0^x f_\gamma(y)dy$, and [*15, Eq. (2.19.5.3)*]. □

TABLE I
LMS CHANNEL PARAMETERS [3]

Shadowing	b	m	Ω
Frequent heavy shadowing (FHS)	0.063	0.739	8.97×10^{-4}
Average shadowing (AS)	0.126	10.1	0.835
Infrequent light shadowing (ILS)	0.158	19.4	1.29

IV. PERFORMANCE ANALYSIS

With the help of the statistics derived in Section III, we now develop a performance analysis of the system.

A. MGF

The MGF of the scheme can be calculated as

$$M_\gamma(s) = E_{\|\mathbf{h}_l\|^2}[e^{-s\gamma}]. \tag{9}$$

From (5) and (9), the MGF can be expressed as

$$\begin{aligned} M_\gamma(s) &= \alpha^N \sum_{l=0}^{c} \binom{c}{l} \beta^{c-l} \int_0^\infty e^{-\frac{sE_x z}{\sigma^2}} \Big(\frac{z^{d-l-1}}{\Gamma(d-l)} \\ &\times \ {}_1F_1(d; d-l; -(\beta-\delta)z) + \frac{\epsilon\delta z^{d-l}}{\Gamma(d-l+1)} \\ &\times \ {}_1F_1(d+1; d-l+1; -(\beta-\delta)z)\Big)dz. \end{aligned} \tag{10}$$

The integral in (10) can be rewritten by using the relation:

$$e^{-\frac{sE_x z}{\sigma^2}} = G_{01}^{10}\Big(\frac{sE_x z}{\sigma^2}\Big|{}^{\cdot}_{0}\Big) \tag{11}$$

and

$$\begin{aligned} &{}_1F_1(d; d-l; -(\beta-\delta)y) \\ &= \frac{\Gamma(d-l)}{\Gamma(d)} G_{12}^{11}\Big((\beta-\delta)y\Big|{}^{1-d}_{0,1-d+l}\Big) \end{aligned} \tag{12}$$

as

$$\begin{aligned} M_\gamma(s) &= \alpha^N \sum_{l=0}^{c} \binom{c}{l} \beta^{c-l} \Big(\int_0^\infty \frac{z^{d-l-1}}{\Gamma(d)} G_{01}^{10}\Big(\frac{sE_x z}{\sigma^2}\Big|{}^{\cdot}_{0}\Big) \\ &\times G_{12}^{11}\Big((\beta-\delta)z\Big|{}^{1-d}_{0,1-d+l}\Big) + \frac{\epsilon\delta z^{d-l}}{\Gamma(d+1)} G_{01}^{10}\Big(\frac{sE_x z}{\sigma^2}\Big|{}^{\cdot}_{0}\Big) \\ &\times G_{12}^{11}\Big((\beta-\delta)z\Big|{}^{-d}_{0,-d+l}\Big)dz. \end{aligned} \tag{13}$$

With the help of [16, Eq. (21)], the integral given in (13) can be solved in closed-form as

$$\begin{aligned} M_\gamma(s) &= \alpha^N \sum_{l=0}^{c} \binom{c}{l} \beta^{c-l} \Big(\frac{(s\bar{\gamma})^{-d+l}}{\Gamma(d)} \\ &\times G_{22}^{12}\Big(\frac{(\beta-\delta)}{s\bar{\gamma}}\Big|{}^{1-d,1-d+l}_{0,1-d+l}\Big) + \frac{\epsilon\delta(s\bar{\gamma})^{-d+l-1}}{\Gamma(d+1)} \\ &\times G_{22}^{12}\Big(\frac{(\beta-\delta)}{s\bar{\gamma}}\Big|{}^{-d,-d+l}_{0,-d+l}\Big), \end{aligned} \tag{14}$$

where $\bar{\gamma} = E_x/\sigma^2$.

B. Average BER

In this subsection, we derive the approximate average BER of the considered MRC based scheme for M-QAM and M-PSK constellations, in closed-form.

1)M-QAM: The instantaneous BER of the M-QAM constellation is given as [17-20]

$$P_e(\gamma) \cong \frac{4}{\log_2 M}\Big(1 - \frac{1}{\sqrt{M}}\Big) \sum_{k=1}^{\sqrt{M}/2} Q\left(a_k\sqrt{\gamma}\right), \tag{15}$$

where $a_k = (2k-1)\sqrt{3/(M-1)}$. Hence, the average BER of the system will be

$$P_e(\bar{\gamma}) = \int_0^\infty P_e(x) f_\gamma(x) dx. \tag{16}$$

From (5), (15), and (16), we get

$$\begin{aligned} P_e(\bar{\gamma}) &= \frac{4\alpha^N}{\log_2 M}\Big(1 - \frac{1}{\sqrt{M}}\Big) \sum_{k=1}^{\sqrt{M}/2} \sum_{l=0}^{c} \binom{c}{l} \beta^{c-l} \\ &\times (\mathcal{I}(k,l,d,\bar{\gamma}) + \epsilon\delta\mathcal{I}(k,l,d+1,\bar{\gamma})), \end{aligned} \tag{17}$$

where

$$\mathcal{I}(k,l,d,\bar{\gamma}) = \frac{(\beta-\delta)^{\frac{l-d}{2}}}{\bar{\gamma}^{\frac{d-l}{2}}\Gamma(d-l+1)} \int_0^\infty x^{\frac{d-l}{2}-1} e^{-\frac{\beta-\delta}{2\bar{\gamma}}x}$$

$$\times Q\left(a_k\sqrt{x}\right)M_{\frac{d+l}{2},\frac{d-l-1}{2}}\left(\frac{\beta-\delta}{\bar{\gamma}}x\right)dx. \tag{18}$$

By using the relation: $Q(x) = (1/2)\mathrm{erfc}(x/\sqrt{2})$ in (18), and then using [15, Eq. (2.19.9.4)], it can be shown that

$$\mathcal{J}(k,l,d,\bar{\gamma}) = \frac{\Gamma(d-l+1/2)}{2\sqrt{\pi}\tilde{a}_k^{2d-2l}\bar{\gamma}^{d-l}\Gamma(d-l+1)} \quad {}_3F_2(d,d-l,$$
$$d-l+1/2; d-l+1, d-l; -\frac{(\beta-\delta)}{\bar{\gamma}\tilde{a}_k^2}), \tag{19}$$

where $\tilde{a}_k = a_k/\sqrt{2}$ and ${}_3F_2(\cdot,\cdot,\cdot;\cdot,\cdot;\cdot)$ represents the generalized Hypergeometric function. The BER of the considered system can be evaluated by using (17) and (19), in closed-form, for M-QAM constellation.

2)M-PSK

For M-PSK constellation, the instantaneous BER is given as [17-20]

$$P_e(\gamma) \cong \frac{2}{max\left(\log_2 M, 2\right)} \sum_{k=1}^{\max(M/4,1)} Q\left(\sqrt{2\gamma}b_k\right), \tag{20}$$

where $b_k = \sin\frac{(2k-1)\pi}{M}$. By using a similar procedure used in Subsection IV-A.1, the average BER of the MRC scheme for M-PSK constellation can be obtained from (20), as

$$P_e(\bar{\gamma}) = \frac{2\alpha^N}{max\left(\log_2 M, 2\right)} \sum_{k=1}^{\max(M/4,1)} \sum_{l=0}^{c} \binom{c}{l}\beta^{c-l}$$
$$\times (\mathcal{J}(k,l,d,\bar{\gamma}) + \epsilon\delta\mathcal{J}(k,l,d+1,\bar{\gamma})), \tag{21}$$

where

$$\mathcal{J}(k,l,d,\bar{\gamma}) = \frac{\Gamma(d-l+1/2)b_k^{2l-2d}}{2\sqrt{\pi}\bar{\gamma}^{d-l}\Gamma(d-l+1)} \quad {}_3F_2(d,d-l,$$
$$d-l+1/2; d-l+1, d-l; -\frac{(\beta-\delta)}{\bar{\gamma}b_k^2}). \tag{22}$$

C. Coding Gain and Diversity Order

Let us consider the expression of the average BER of the M-PSK constellation given in (21) and (22) for diversity order calculation. Let us assume that the value of $\bar{\gamma}$ is very high; by observing the fact that ${}_3F_2(c_1,c_2,c_3;d_1,d_2;-z) \to 1,\ z \to 0$ [22], from (21) and (22), we get the following asymptotic BER of the considered scheme:

$$P_e(\bar{\gamma}) = \frac{\alpha^N}{\sqrt{\pi}\,max\left(\log_2 M, 2\right)} \sum_{l=0}^{c} \binom{c}{l}\beta^{c-l}\frac{b_1^{2l-2d}}{\bar{\gamma}^{d-l}}$$
$$\times \frac{\Gamma(d-l+1/2)}{\Gamma(d-l+1)} \underbrace{(1+\frac{\epsilon\delta(d-l+1/2)}{\bar{\gamma}b_1^2(d-l+1)})}_{A}. \tag{23}$$

For very large $\bar{\gamma}$, in (1), the term $A \to 1$; therefore, we get

$$P_e(\bar{\gamma}) = \frac{\alpha^N}{\sqrt{\pi}\,max\left(\log_2 M, 2\right)} \sum_{l=0}^{c} \binom{c}{l}\beta^{c-l}\frac{b_1^{2l-2d}}{\bar{\gamma}^{d-l}}$$
$$\times \frac{\Gamma(d-l+1/2)}{\Gamma(d-l+1)}. \tag{24}$$

At very high SNR, the decay of BER is dominated by the highest power of $\bar{\gamma}$; therefore, we get the following asymptotic BER from (24):

$$P_e(\bar{\gamma}) = \frac{\alpha^N\Gamma(N+1/2)b_1^{-2N}}{\sqrt{\pi}\,max\left(\log_2 M, 2\right)\Gamma(N+1)\bar{\gamma}^N}. \tag{25}$$

At high SNR, the average BER of the communication system is given by $P_e(\bar{\gamma}) = (G_c\bar{\gamma})^{-G_d}$, where G_c is coding gain and G_d denotes diversity order. Therefore, from (25), we have

$$G_d = N$$

$$G_c = \frac{b_1^2}{\alpha}\left(\frac{\sqrt{\pi}\, max\ \left(\underset{2}{log}\, M\, , 2\right)\Gamma(N+1)}{\Gamma(N+1/2)}\right)^{1/N}. \tag{26}$$

D. Probability of Outage

The probability of outage is defined as

$$P_{\text{out}}(\gamma_{\text{th}}) \triangleq \Pr(\gamma \le \gamma_{\text{th}}), \tag{27}$$

where$\Pr(\cdot)$ denotes the probability. From (7) and (27), we get

$$P_{\text{out}}(\gamma_{\text{th}}) = \alpha^N \sum_{l=0}^{c} \binom{c}{l} \beta^{c-l}$$
$$\times (\mathcal{G}(x, l, d, \gamma_{\text{th}}) + \epsilon\delta\mathcal{G}(x, l, d+1, \gamma_{\text{th}})). \tag{28}$$

E. Capacity

The average capacity (in bits/second/Hz) of the MRC based scheme is given by [23,24]

$$C(\bar{\gamma}) = \int_0^\infty \underset{2}{\log}(1 + x\bar{\gamma}) f_{||\mathbf{h}||^2}(x) dx. \tag{29}$$

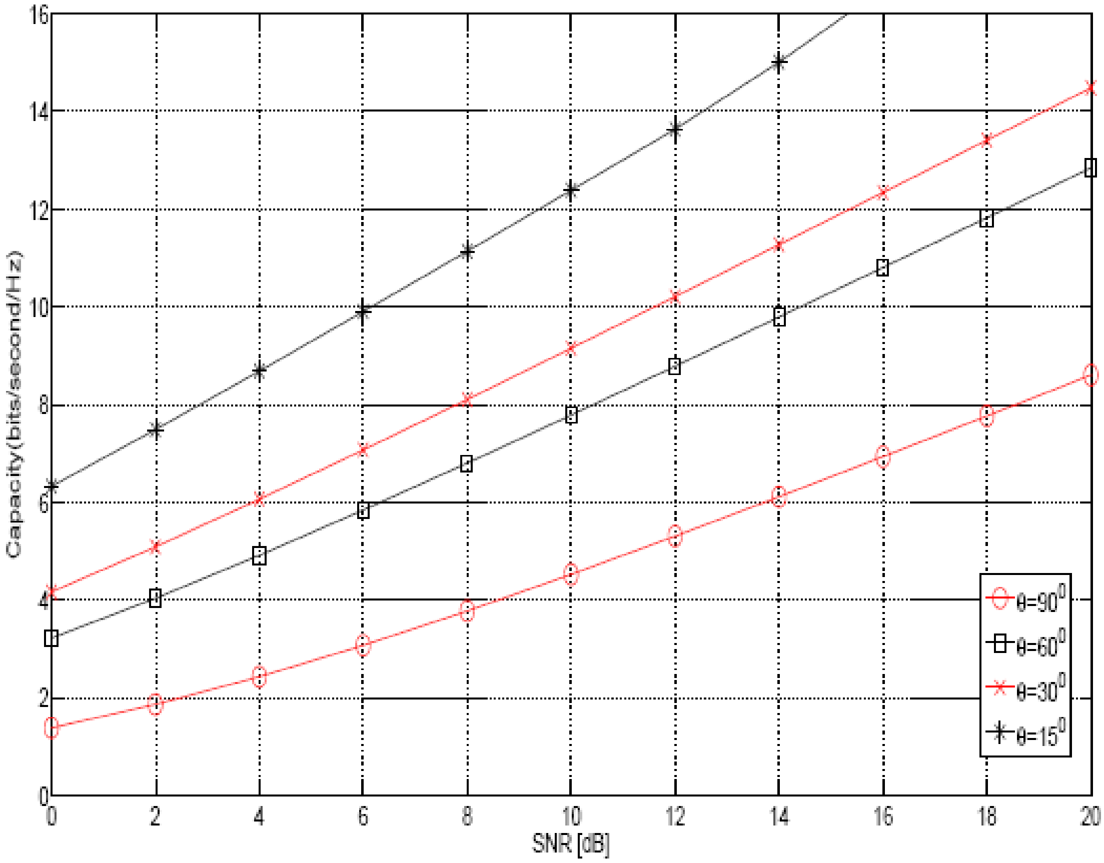
We can represent $\ln(1+x)$ and $_1F_1(d; d-l; -(\beta-\delta)x)$, where $_1F_1(\cdot;\cdot;\cdot)$ denotes the confluent Hypergeometric function, in the form of Meijer-G function, through [15, Eqs. (8.4.6.5) and (8.4.45.1)]. By using these representations, (29), (5), [16, Eq. (21)],,and after some algebra, the capacity of the MRC scheme can be expressed as

$$C(\bar{\gamma}) = \alpha^N \sum_{l=0}^{c} \binom{c}{l} \frac{\beta^{c-l}}{ln\ 2} (\mathcal{K}(l, d, \bar{\gamma}) + \epsilon\delta\mathcal{K}(l, d+1, \bar{\gamma})). \tag{30}$$

In (8), $\mathcal{K}(l, d, \bar{\gamma})$ is given by

$$\mathcal{K}(l, d, \bar{\gamma}) = \frac{1}{\bar{\gamma}^{d-l}\Gamma(d)}$$
$$\times G_{3,4}^{3,2}\left(\frac{\beta-\delta}{\bar{\gamma}} \middle| \begin{matrix} 1+l-d, l-d, 1+l-d \\ 0, l-d, l-d, 1+l-d \end{matrix}\right), \tag{31}$$

where$G_{p,q}^{m,n}(\cdot \mid \begin{smallmatrix} \cdots \\ \cdots \end{smallmatrix})$ is the Meijer-G function.



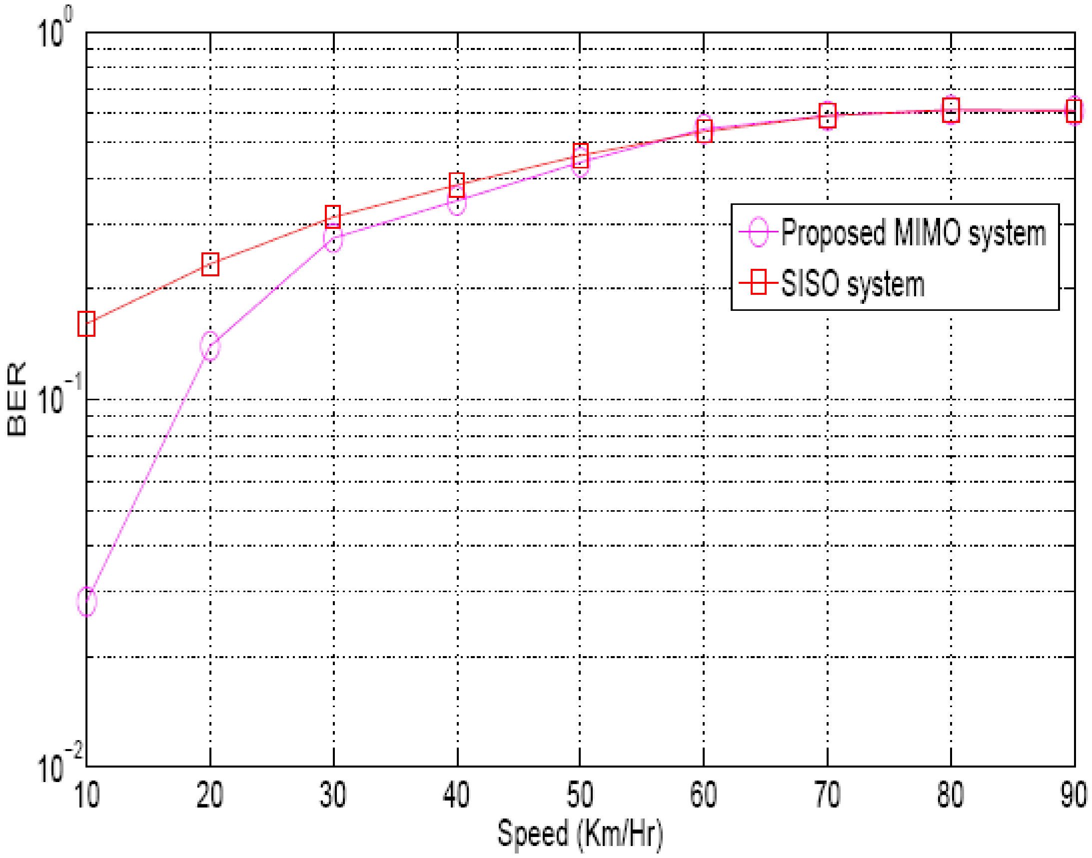
1. Capacity of the polarization based scheme with different elevation angles.

V. NUMERICAL RESULTS AND CONCLUSIONS

The analytical capacities (in bits/second/HZ) of the considered polarization based scheme are plotted in Fig. 1 with elevation angles equal to $15^0, 30^0, 60^0$,and 90^0. The expression of elevation angle is taken from [3]. It can be seen from Fig. 1 that the average capacity of the system is increased with increasing values of elevation angles.

Fig. 2 shows the simulated BER with QPSK constellation for SISO system and proposed scheme for different speed of mobile vehicle. It can be observed from Fig. 2, that proposed scheme outperform the SISO system, especially when the speed of mobile vehicle is low.If speed is high, i.e., 50 to 90 Km/Hr, no improvement is observed.

The simulated and analytical capacity versus SNR plots of the considered MRC scheme are plotted in Fig. 3 for $N = 2,3,4$ and under frequent heavy shadowing. The tight matching of the proposed closed-form capacity expression given in (8) and simulated capacity results is evident from the figure. As seen in Fig. 3 that by increasing the receiving antennas from two to four, a capacity gain of approximately 1.2 bits/second/Hz can be achieved at SNR=20 dB under heavy shadowing.



2. Simulated BER of the polarization based scheme with different speed of mobile vehicle

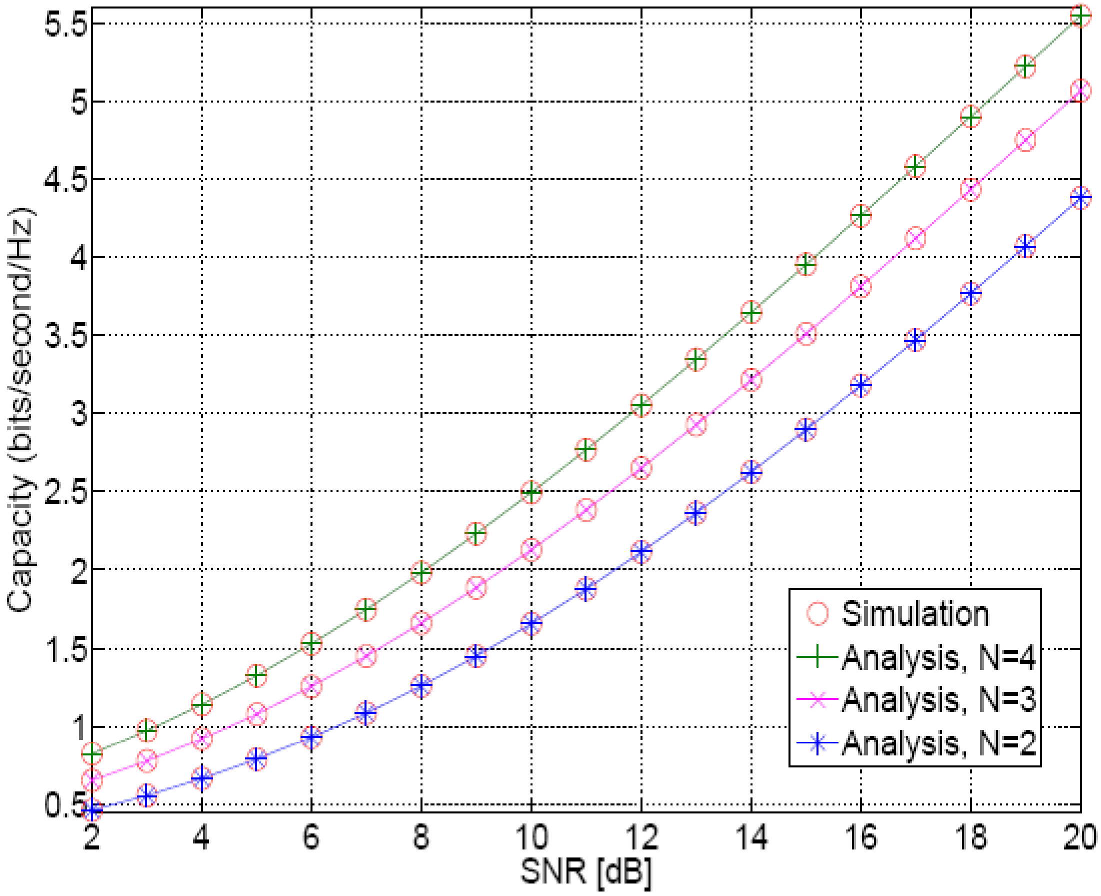


Fig. 3. Analytical and simulated capacity of the MRC scheme with N = 2, 3, 4 and FHS.

REFERENCES

[1] P. Chini, G. Giambene, and S. Kota, "*A survey on mobile satellite system s*," *Int. J. Sat. Commun.*, vol. 28, no. 1, pp. 29–57, Aug. 2009.

[2] P. Arapoglou, K. Liolis, M. Bertinelli, A. Panagopoulos, P. Cottis, and R. De Gaudenzi, "*MIMO over satellite: A review*," *IEEECommun. Surveys Tut.*, vol. 13, no. 1, pp. 27–51, May 2011.

[3] A. Abdi, W. Lau, M.-S. Alouini, and M. Kaveh, "*A new simple model f or land mobile satellite channels: First and second order statistics*," *IEEE Trans. Wireless Commun.*, vol. 2, no. 3, pp. 519–528, May 2003.

[4] G. Alfano and A. D. Maio, "*Sum of squared Shadowed-Rice rando m variables and its application to communication systems performance prediction*," *IEEE Trans. Wireless Commun.*, vol. 6, no. 10, pp. 3540–3545, Oct. 2007.

[5] G. Ropokis, A. Rontogiannis, K. Berberidis, and P. Mathiopoulos, " *Performance analysis of maximal ratio combining over Shadowed-Rice fading channels,*" *International Workshop on Satellite and Space Communications (IWSSC 2009)*, pp. 83–87, Sep. 2009, Tuscany, Italy.

[6] G. Ropokis, A. Rontogiannis, P. Mathiopoulos, and K. Berberidis, "*An exact performance analysis of MRC / OSTBC over generalized fading channels*," *IEEE Trans. Commun.*, vol. 58, no. 9, pp. 2486–2492, Sep. 2010.

[7] M. R. Bhatnagar, "*On the sum of correlated squared $\kappa-\mu$ shadowed random variables and its application to performance analysis of MRC*," *IEEE Trans. Vehicular Technol.*, pp. 1–7, July 2014, in press.

[8] M. R. Bhatnagar and Arti M.K., "*Performance analysis of AF bas ed hybrid satellite-terrestrial cooperative network over generalized fading channels*," *IEEE Commun. Lett.*, vol. 17, no. 10, pp. 1912–1915, Oct. 2013.

[9] M. R. Bhatnagar and Arti M.K., "*On the closed-form performance analysis of maximal ratio combining in Shadowed-Rician fading LMS channels*,"

IEEE Commun.Lett., vol. 18, no. 1, pp. 54–57, Jan. 2014.

[10] M. R. Bhatnagar, "*Performance evaluation of decode-and-forward satellite relaying*," *IEEE Trans. Vehicular Technol.*, pp. 1–7, Nov. 2014, in press.

[11] Arti M.K. and M. R. Bhatnagar, "*Performance analysis of hop- by-hop beamforming and combining in DF MIMO relay system over Nakagami-M fading channels*," *IEEE Commun. Lett.*, vol. 17, no. 11, pp. 2080–2083, Nov. 2013.

[12] Arti M.K. and M. R. Bhatnagar, "*Beamforming and combining in hybrid satellite-terrestrial cooperative systems*," *IEEE Commun. Lett.*, vol. 18, no. 3, pp. 483–486, March 2014.

[13] Arti M. K. and M. R. Bhatnagar, "*Two-way mobile satellite relaying: A beamforming and combining based approach*," *IEEE Commun.Lett.*, vol. 18, no. 7, pp. 1187 - 1190, July 2014.

[14] E. G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*. Cambridge, UK: Cambridge University Press, 2003.

[15] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series*, 1st ed. New York, USA: Gordon and Breach Science Publishers, 1990, vol. 3.

[16] V. S. Adamchik and O. Marichev, "*The algorithm for calculating integrals of hypergeometric type function and its realization in reduce system*," in *Proc. International Symposium on Symbolic Algebraic Computation (ISSAC'90)*, Tokyo, Japan, Aug. 1990, pp. 212–224.

[17] J. Lu, K. B. Letaief, J. C.-I. Chuang, and M. L. Liou, "*M -PSK and M -QAM BER computation using signal-space concepts*," *IEEETrans. Commun.*, vol. 47, no. 2, pp. 181–184, Feb. 1999.

[18] M. R. Bhatnagar, "*Performance Analysis of A Path Selection Scheme in Multi-hop Decode-and-Forward Protocol*," *IEEE Commun.Lett.*, vol. 16, no. 12, pp. 1980 - 1983, Dec. 2012.

[19] M. R. Bhatnagar, "*Performance Analysis of Max-Min Path Selection Scheme in Multi-Hop DF Cooperative System Over Nakagami-m Channels*," In Proc. for SPCOM 2014, Bangalore, India, July 2014.

[20] Arti M.K. and M. R. Bhatnagar, "*Maximal Ratio Transmission in AF MIMO Relay Systems over Nakagami-m Fading Channels*," IEEE Trans. on Vehicular Technol., vol. PP, no 99, pp. 1, July 2014, DOI: 10.1109/TVT.2014.2334631.

[21] M. R. Bhatnagar and Arti M.K., "*Performance Analysis of Hybrid Satellite-Terrestrial FSO Cooperative System,"* IEEE Photonics Technology Lett., vol. 25, no. 22, pp. 2197 - 2200, Nov. 2013.

[22] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. New York, USA: Dover Publications, Inc., 1972.

[23] M. R. Bhatnagar, "*On the Capacity of Decode-and-Forward Relaying over Rician Fading Channels*," IEEE Commun. Lett., vol. 17, no. 6, pp. 1100 - 1103, June 2013.

[24] M. R. Bhatnagar and S. Anees "*On the Performance of Alamouti Scheme in Gamma-Gamma Fading FSO Links with Pointing Errors*," IEEE Wireless Commun.Lett., vol. 4, no. 1, pp. 94-97, Feb. 2015.

[25] Arti M.K., Ranjan K. Mallik, and Robert Schober, "*Beamforming and Combining in Two-Way AF MIMO Relay Networks*," *IEEE* Commun.Lett.,, volume 17, number 7, pages 1400–1403, July 2013.

[26] Arti M.K., Ranjan K. Mallik, and Robert Schober, "*Performance Analysis of OSTBC Relaying in AF MIMO Relay System with Imperfect CSI," IEEE Transactions on Vehicular Technology*, in press.

[27] Arti M.K., Ranjan K. Mallik, and Robert Schober, "*Channel Estimation and Decoding of OSTBC in Two-Way AF MIMO Relay Networks*," in *Proceedings of the IEEE Vehicular Technology Conference (VTC) - Fall*, Las Vegas, USA, 2-5 September 2013.

[28] Arti M.K., Ranjan K. Mallik, and Robert Schober, "*Joint Channel Estimation and Decoding of Space-Time Block Codes in AF MIMO Relay Networks*," in *Proceedings of the International Conference on Signal Processing and Communications (SPCOM)*, Bangalore, India, 22-25 July 2012, pp. 1–5.

[29] Arti M.K., Ranjan K. Mallik, and Robert Schober, "*Beamforming and Combining Based on Estimated Channels in Cooperative Relay Networks,"* in *Proceedings of the National Conference on Communications (NCC)*, Kharagpur, India, 3-5 February 2012, pp. 1–5.

[30] Manav R. Bhatnagar and Arti M.K. "*Selection Beamforming and Combining in Decode-and-Forward MIMO Relay Networks*," *IEEE Communications Letters*, volume 17, number 8, pages 1556 - 1559, August 2013.